Université de Fribourg — Département de Mathématiques
Universität Freiburg — Mathematik Departement

Mathematical Methods for Computer Science I

Fall 2016

Series 9 – Hand in before Monday, 28.11.2016 - 12.00

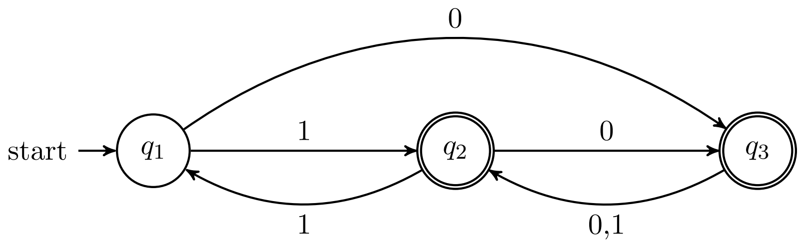
1. Let L be a language. The aim of this exercise is to prove that *if L is accepted by an ε-NFA, then L is accepted by an NFA.*
   To this aim, carry out the following steps. As a preparation, suppose that $E = (Q, \Sigma, \delta, q_0, F)$ is the ε-NFA that recognizes L. Set $N = (Q, \Sigma, \delta', q_0, F')$, an NFA with:
   $$F' = \begin{cases} F \cup \{q_0\} & \text{if } \mathcal{C}(q_0) \cap F \neq \emptyset \\ F & \text{otherwise} \end{cases}$$
   $$\delta'(q, a) = \hat{\delta}(q, a) = \mathcal{C}(\delta(\mathcal{C}(q), a)) \ \forall q \in Q, \ a \in \Sigma.$$
   a) Prove by induction on $|x|$, where $x \in \Sigma^*$, that $\delta'(q_0, x) = \hat{\delta}(q_0, x)$.
   b) Prove that $\delta'(q_0, x) \cap F' \neq \emptyset \iff \hat{\delta}(q_0, x) \cap F \neq \emptyset$, that is, the language L accepted by the ε-NFA is accepted by the constructed NFA.
   Hint: distinguish the cases $x = \varepsilon$ and $x \neq \varepsilon$.

2. Give English descriptions of the languages of the following regular expressions
   a) $(b + \varepsilon)(aa^*b)^*(aa)^*$.
   b) $(a + b)(b + a)^*bbbb(a^*b^*)^*$.
   c) $a^*(a + abb)^*a^*$.

3. Write regular expressions for the following languages
   a) The language of words over the alphabet $\Sigma = \{a, b, c\}$ containing at least one a and at least one b.
   b) * The language of words over the alphabet $\Sigma = \{a, b\}$ in which every subword aa appears after all the word's subwords bb, including words without aa or bb.

4. Convert the regular expression $R = 00(0 + 1)^* + 1$ into an ε-NFA over the binary alphabet $\{0, 1\}$.

5. Convert the following automaton into a regular expression.



* Exercises with a $\star$ are intended for Discrete Mathematics I students only. However, MMI I students can gain additional bonus points by attempting them.