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## Mathematical Methods for Computer Science II

Spring 2017

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Series 8 – Hand in before Monday, 24.04.2017 - 13.00

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1. Determine which of the following are logical equivalences either by simplifying or by writing truth tables.

- a)  $p \Rightarrow q \stackrel{?}{\equiv} \neg q \Rightarrow \neg p$
- b)  $\neg(p \Leftarrow q) \stackrel{?}{\equiv} \neg q \wedge p$
- c)  $(p \wedge q) \Rightarrow r \stackrel{?}{\equiv} q \Rightarrow (r \Rightarrow p)$
- d)  $p \Rightarrow (q \Rightarrow r) \stackrel{?}{\equiv} q \Rightarrow (p \Rightarrow r)$
- e)  $(p \wedge q) \Rightarrow r \stackrel{?}{\equiv} (p \Rightarrow r) \vee (q \Rightarrow r)$
- f)  $(p \wedge q) \Rightarrow r \stackrel{?}{\equiv} (p \Rightarrow r) \wedge (q \Rightarrow r)$
- g)  $(p \vee q) \Rightarrow r \stackrel{?}{\equiv} (p \Rightarrow r) \vee (q \Rightarrow r)$
- h)  $p \Rightarrow (q \vee r) \stackrel{?}{\equiv} (p \Rightarrow q) \vee (p \Rightarrow r)$ .

2. We say that  $p \oplus q$ , where the logical operator  $\oplus$  is called the *exclusive disjunction* or *xor*, is true if and only if exactly one of  $p$  and  $q$  is true and the other is false.

Write down a truth table for  $p \oplus q$  and show the following equivalences :

- a)  $p \oplus \neg p \equiv T$
- b)  $p \oplus p \equiv F$
- c)  $p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$
- d)  $p \oplus q \equiv \neg(p \wedge q) \wedge (p \vee q)$ .

3. Which of the following formulas are tautologies (i.e. always true) ?

- a)  $((p \Rightarrow q) \Rightarrow q) \Rightarrow p$
- b)  $((p \Rightarrow q) \Rightarrow p) \Rightarrow p$
- c)  $(p \Rightarrow r) \Rightarrow (p \vee q \Rightarrow r)$
- d)  $(p \Rightarrow r) \Rightarrow (p \wedge q \Rightarrow r)$
- e)  $(p \vee q \Rightarrow r) \Rightarrow (p \Rightarrow r)$
- f)  $(p \wedge q \Rightarrow r) \Rightarrow (p \Rightarrow r)$

In case one of the formulas can be false, give an example of values of  $p, q, r$  such that the formula is false.

4. Explain what is wrong with the following argument.

“If it rains, we are not going for a walk. Since it does not rain, we are going for a walk.”

5. Consider the following statements:

- a) Everybody having a musical ear is able to sing properly.
- b) Nobody is a real musician if they cannot electrify their audience.
- c) Nobody who does not have a musical ear can electrify their audience.
- d) Nobody, except a real musician, can compose a symphony.

Question: Which properties does a person have who has composed a symphony?

*Hint:* Formalize these assertions, and use truth-tables.