
Mathematical Methods for Computer Science II

Spring 2017

Series 7 – Hand in before Monday, 10.04.2017 - 13.00

1. a) Suppose there are 60 clothing items in a wardrobe: 20 are white, 15 are shirts, 40 are made of cotton, 7 are white shirts, 13 are cotton shirts, 10 are white and made of cotton, and 5 are white cotton shirts. How many clothing items are there neither white, nor shirts, nor made of cotton?
b) Use the *inclusion-exclusion principle* to determine the number of positive integers between 1 and 2500 which are either perfect squares, or perfect cubes or both.
2. a) Imagine you have prepared 8 personalised emails for 8 distinct people. How many possibilities are there to send all the emails to wrong addressees?
b) Show that $D(n)$, the number of derangements of a list of n elements, has the following recursive formula:

$$D(n) = (n - 1)(D(n - 1) + D(n - 2)), \quad \text{with } D(1) = 0 \text{ and } D(2) = 1.$$

3. Use the *inclusion-exclusion principle* to find a formula for the the number of ways of seating n couples (that is $2n$ people in total) around a round table such that at least one couple are sitting together.
Hint: take A to be the set of all seating plans. For $i = 1, \dots, n$ let P_i be the property “couple i sit together”.
4. * Two integers are said to be relatively prime if 1 is their greatest common divisor. If n is a positive integer, let $\varphi(n)$ be the number of positive integers $m \leq n$ that are relatively prime to n . This function is called the *Euler φ function*.

a) Show that for any prime number p and any integer a ,

$$\varphi(p^a) = p^a - p^{a-1}.$$

b) Find a formula for $\varphi(pq)$, where p and q are distinct prime numbers.

c) Suppose n is divisible by precisely r different primes, p_1, \dots, p_r . Prove the following formula by using the *inclusion-exclusion principle*

$$\varphi(n) = n \prod_{i=1}^r \left(1 - \frac{1}{p_i}\right).$$

* Exercises with a * are intended for Discrete Mathematics II students only. However, MMI II students can gain additional bonus points by attempting them.