Mathematical Methods for Computer Science II

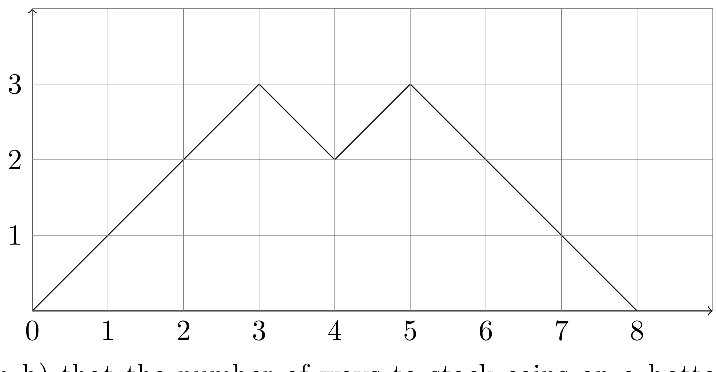
Spring 2017

Series 6 – Hand in before Monday, 03.04.2017 - 13.00

1. Consider the following product
$$((((a(bc))d)e)((f(gh))i))j$$
   a) Give the binary tree corresponding to it.
   b) Give the polygon triangulation corresponding to it.

2. Prove the recursive formula of the Catalan numbers c_n seen in class
$$c_n = \begin{cases} 0 & n = 0 \\ 1 & n = 1 \\ \sum_{k=1}^{n-1} c_k c_{n-k} & n \geq 2 \end{cases}$$
using only the fact that the number of triangulations of a convex polygon with $n+1$ vertices ($n \geq 2$) is the Catalan number c_n.
Hint: To get an idea of a possible proof suing recurrence, consider first a hexagon (convex polygon with 6 vertices) and start with drawing all possible configurations of a triangle based on the horizontal side at the top of the hexagon. The for each configuration, count the number of triangulations. Once this mechanism of the proof is understood in the case of the hexagon, generalise to an arbitrary polygon to conclude the proof.

3. a) Consider paths (or mountain ridgelines) in the (x, y)-plane from $(0, 0)$ to $(2n, 0)$ with steps $(1, 1)$ and $(1, -1)$ that never pass below the x-axis as the one depicted below. For $n = 0, 1, 2, 3, 4$, illustrate all the possible mountain ridgelines.
   b) Show that for any n, the number possible mountain ridgelines is equal to c_{n+1}, where c_n is the n-th Catalan number.



   c) Deduce from b) that the number of ways to stack coins on a bottom row that consists of n consecutive coins in the plane, such that no coins are allowed to be put on the two sides of the bottom coins and every additional coin must be above two other coins, is equal to c_{n+1}.