Mathematical Methods for Computer Science I

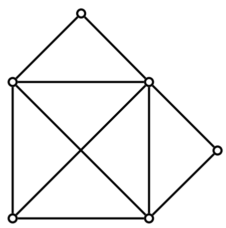
Fall 2016

Series 6 – Hand in before Monday, 7.11.2016 - 13.00

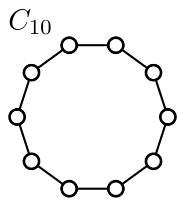
1. **Definition**: Given a planar representation of a graph G, a **region** is a maximal section of the plane in which any two points can be joined by a curve that does not intersect any part of G.
   Prove *Euler's formula*:
   If G is a connected planar graph with n vertices, q edges, and r regions, then
   $$n - q + r = 2.$$
   Hint: Induction over q; treat trees and any other graphs separately. To treat the latter, think about what happens if an edge is removed.

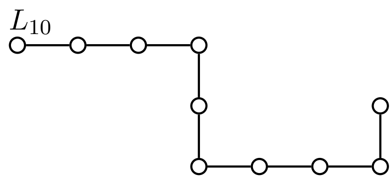
2. Let G be a connected planar graph with n vertices, q edges, and r regions.
   Definition: For a region r in G, the **bound degree**, denoted by $b(r)$, is the number of edges that bound region r.
   a) Illustrate the above definition with an example.
   b) Illustrate Euler's formula with an example.
   c) Consider the sum of the bound degrees of G,
   $$C(G) = \sum_r b(r).$$
   Prove that $C(G) \leq 2q$.
   d) Prove that $C \geq 3r$.
   e) Prove that if $n \geq 3$, then $q \leq 3n - 6$.
   Furthermore, if equality holds, then every region is bounded by three edges.
   f) Prove that K_5 is not planar.
   g) Prove that $\delta(G) \leq 5$.
   Hint: Use Euler's formula.

3. Find the chromatic polynomial and chromatic number of the following graph.



4. Let C_n be the circuit graph with n vertices and L_n a line graph with n vertices:





   a) Find a recursive formula for $p(L_n, \lambda)$. [Hint: recursively decompose L_n.]
   b) Use a) to compute $p(L_n, \lambda)$ explicitly.
   c) Prove that
   $$p(C_n, \lambda) = (\lambda - 1)^n + (-1)^n(\lambda - 1).$$
   [Hint: decompose C_n, use the formula found in b) and apply induction.]