
Mathematical Methods for Computer Science II

Spring 2017

Series 5 – Hand in before Monday, 27.03.2017 - 13.00

1. Find the generating functions of each of the following sequences, in simple, closed form. In each case the sequence is defined for all $n \geq 0$.

- a) $a_n = n$
- b) $a_n = \alpha n + \beta$
- c) $a_n = \alpha n^2 + \beta n + \gamma$
- d) $a_n = 5 \cdot 7^n - 3 \cdot 4^n$

2. Solve the following recursive equations using the generating function method for a) and the ansatz method for b).

a)

$$a_k = \begin{cases} 5 & k = 0 \\ 3 & k = 1 \\ 5k^2 - 6k - 4 + a_{k-2} & k \geq 2. \end{cases}$$

b)

$$b_k = \begin{cases} 0 & k = 0 \\ 1 & k = 1 \\ -3b_{k-1} - 2b_{k-2} + k^2 & k \geq 2. \end{cases}$$

Hint for b): particular solution of the inhomogeneous equation $ak^2 + bk + c$.

3. The Fibonacci numbers $(F_n)_{n \in \mathbb{N}}$ are recursively defined by

$$F_n = \begin{cases} 1 & n = 0, 1 \\ F_{n-1} + F_{n-2} & n \geq 2. \end{cases}$$

Derive an explicit formula for the Fibonacci numbers F_n using the

- a) generating function method.
- b) ansatz method.

4. The growth of rabbit population may be modelled in the following way :

- a population of N adult rabbits gives birth to N baby rabbits every month,
- a rabbit is adult at the age of 1 month, it can have its first baby rabbit at the age of 2 months.

A farmer has A newborn rabbits. At the end of the first month, he buys $B - A$ newborn rabbits, so that he then has a total of B rabbits. Denote by E_n the number of rabbits at the farm on the first day of the n th month.

- a) Find a recursive relation satisfied by the E_n .
- b) Compute E_n in terms of A , B and the Fibonacci numbers F_n .
- c) * Identify E_n when $A = F_a$ and $B = F_{a+1}$ for some $a \geq 0$.
- d) * Deduce that $F_{a+b} = F_a F_b + F_{a-1} F_{b-1}$ for any $a, b \geq 1$.

5. Let $f(n)$ be the number of subsets of $\{1, \dots, n\}$ that contain no two consecutive elements, for integer n . Find the recursive equation that is satisfied by these numbers, and then find the numbers $f(n)$ themselves.

Hint: to find the recursive equation, separate the subsets into two classes, those that contain the integer n and those that do not.

* Exercises with a * are intended for Discrete Mathematics II students only. However, MMI II students can gain additional bonus points by attempting them.