

Mathematical Methods for Computer Science I

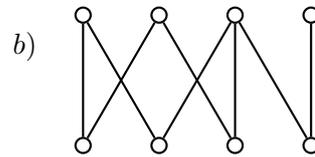
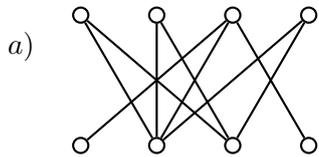
Fall 2016

Series 5 – Hand in before Monday, 24.10.2016 - 13.00

1. *** Definition:** A set C of vertices in a graph G is said to **cover** the edges of G if every edge of G is incident with at least one vertex of C . Such a set C is called an **edge cover** of G .

Prove that in a bipartite graph G , the maximum number of edges in a matching in G equals the minimum number of vertices in an edge cover of G .

2. Using Hall's Theorem determine whether or not the following graphs have a perfect matching.



3. **Definition:** A k -regular graph is a graph where all the vertices have degree k .

Let G be a k -regular ($k \geq 1$) bipartite graph. Prove that G has a perfect matching. Does the result remain true if the assumption that G is bipartite is removed?

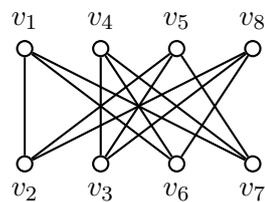
4. Senate committees C_1 through C_7 consist of the members as indicated:

$C_1 = \{\text{Eggers, Bradford, Gardner}\}$, $C_2 = \{\text{Adams, Bradford, Charles}\}$, $C_3 = \{\text{Davis, Ford}\}$, $C_4 = \{\text{Howe, Charles, Ford}\}$, $C_5 = \{\text{Eggers, Howe}\}$, $C_6 = \{\text{Charles, Davis, Eggers}\}$, $C_7 = \{\text{Adams, Gardner}\}$.

Determine how the committee meetings have to be scheduled so that each member can attend all the meetings of his committee. To save time, the aim is to schedule as many meetings in parallel as possible.

Hint: Construct a meaningful graph and solve the problem using vertex colouring.

5. a) Use the greedy colouring algorithm to find a colouring of the following graph using the vertex ordering shown.



- b) Use the improvement of the greedy algorithm to color the above graph using only $\chi(G)$ colours.
 c) Prove that any graph G has a vertex ordering for which the greedy algorithm uses only $\chi(G)$ colours.
 d) Inspired by a), explain why for each $n > 1$ one can construct a bipartite graph with $2n$ vertices for which the greedy algorithm uses n colours.

* Exercises with a \star are intended for Discrete Mathematics I students only. However, MMI I students can gain additional bonus points by attempting them.