Université de Fribourg
Universität Freiburg

Département de Mathématiques
Mathematik Departement

Mathematical Methods for Computer Science I

Fall 2016

Series 4 – Hand in before Monday, 17.10.2016 - 13.00

1. **Definition:** A vertex v in a connected graph G is a *cut-vertex* if $G \backslash v$ is disconnected. Prove the following two statements:
   a) Every nontrivial connected graph contains at least two vertices that are not cut-vertices.
   b) A vertex v in a graph G is a cut-vertex of G if and only if there are two vertices u and w distinct from v such that v lies on every path from u to w in G.

2. * **Definition:** A nontrivial connected graph containing no cut-vertices is a *nonseparable graph*.
   Let G be a nonseparable graph with at least 3 vertices. Prove that every two vertices of G lie on a common cycle of G.

3. Let $G = (V, E)$ be a graph with $|V| = 2n$ and $\delta(G) \geq n$. Prove that G has a perfect matching.

4. How many perfect matchings are there
   a) in the complete graph K_n?
   b) in a tree?

5. The mathematical institute offers six lectures. Each of the seven members of the institute, A, B, C, D, E, F, G, agrees to give certain lectures as illustrated in the table below.

professor	lectures
A	analysis, numerics
B	analysis, linear algebra
C	analysis, number theory, discrete maths
D	numerics, linear algebra, discrete maths, graph theory
E	graph theory, numerics
F	analysis, linear algebra, discrete maths
G	linear algebra, number theory

   a) Can you assign the professors to the lectures so that none has to teach more than one lecture? To solve this exercise, apply an algorithm seen in class.
   b) Write a program in your preferred programming language that finds augmenting paths in a bipartite graph. Test it on the example of this exercise.

* Exercises with a $\star$ are intended for Discrete Mathematics I students only. However, MMI I students can gain additional bonus points by attempting them.