Mathematical Methods for Computer Science II

Spring 2017

Series 3 – Hand in before Monday, 13.03.2017 - 13.00

1. Let m, n, k, i be nonnegative integers such that $k \leq m + n$ and $i \leq m$. We want to prove the following equality:

$$\binom{m+n}{k} = \sum_{i=0}^{k} \binom{m}{i}\binom{n}{k-i}$$

   This can be done algebraically but also with a combinatorial argument, which is the aim of this exercise.
   a) How many subsets with k elements of $\{1, 2, \ldots, m+n\}$ are there such that exactly i elements are in $\{1, 2, \ldots, m\}$?
   b) Deduce the above equality.

2. a) In how many ways can we rearrange the word "subconsciousness"?
   b) Among these words, how many keep the *order* of the vowels of the word "subconsciousness", but not necessarily their positions?

3. Using generating functions, find the number of ways of collecting 50 CHF from 15 distinct people if the first person can give 1 CHF, 8 CHF or nothing and each of the remaining 14 people can give either 1 CHF or 5 CHF. Interpret the result.

4. Let

$$f(z) = \sum_{k=0}^{n} \binom{n}{k} z^k.$$

   a) Describe the coefficients of $f(z)^2$.
   b) Express $f(z)$ and $f(z)^2$ as powers of $(1 + z)$.
   c) Using a) and b), prove that

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}.$$

5. $\star$
   a) Given a positive integer k, determine the number of positive solutions $(x_1, x_2, x_3, x_4) \in \mathbb{N}^4_{>0}$ of the equation $x_1 + x_2 + x_3 + x_4 = k$.
   b) Given a positive integer k, determine the number of nonnegative solutions $(x_1, x_2, x_3, x_4) \in \mathbb{N}^4$ of the inequality $x_1 + x_2 + x_3 + x_4 \leq k$.

$\star$ Exercises with a $\star$ are intended for Discrete Mathematics II students only. However, MMI II students can gain additional bonus points by attempting them.