Université de Fribourg
Universität Freiburg

Département de Mathématiques
Mathematik Departement

Mathematical Methods for Computer Science II

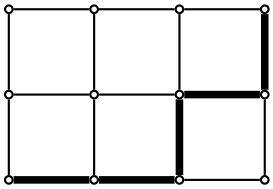
Spring 2017

Series 2 – Hand in before Monday, 06.03.2017 - 13.00

1. Find the solutions to the problems below and explain.
   a) Determine the number of ways to distribute
      i) n balloons of the same color to k children.
      ii) n balloons of different colors to k children.
   b) Determine the number of ways to distribute
      i) n balloons of the same color to k children so that each child gets at least one.
      ii) n balloons of different colors to k children so that each child gets at least one.
   c) We distribute n balloons of the same color to k boys and ℓ girls, in such a way that each of the girls gets at least one balloon, but we do not require the same thing for the boys.
      i) In how many ways can we do this?
      ii) Same as i) but with balloons of different colors.

2. Keys are made by cutting slits of various depths into a piece of metal. Suppose you have a machine which can cut slits of 9 different depths. With how many slits do you have to design the keys such that they all have the same number of slits and that you can produce at least 4 million distinct keys?

3. A robot starts a walk at the bottom left of an $m \times n$ grid. He can only move to the right and toward the top. In how many distinct ways can he reach the the top right end of the grid? The picture below shows a possible path on the 3×4 grid.



4. A large group of n friends intends to work in 5 libraries. Each person goes to exactly one library. Find the number of possibilities for the friends to go to the libraries under each of the following different assumptions. Explain.
   a) The order in which they enter each library matters.
   b) Only the number of friends who goes to each library matters. Neither the order nor who goes to which library matters.
   c) Same as above, except that it does matter who goes to each library.

5. * Let $0 \le k \le m \le n$. Prove the following identity

$$\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}$$

   a) with an algebraic proof by expanding the binomial coefficients.
   b) with a combinatorial proof.

* Exercises with a $\star$ are intended for Discrete Mathematics II students only. However, MMI II students can gain additional bonus points by attempting them.