Université de Fribourg
Universität Freiburg

Département de Mathématiques
Mathematik Departement

Mathematical Methods for Computer Science I

Fall 2016

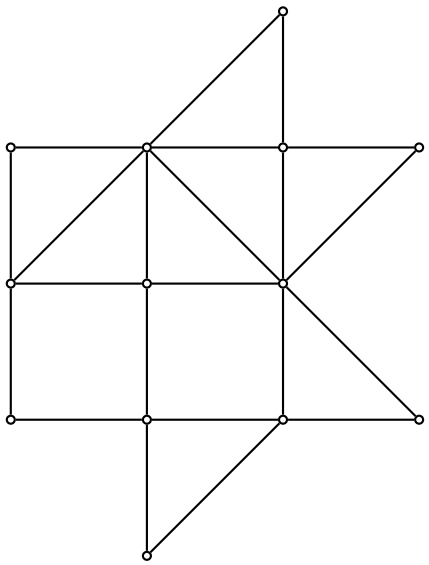
Series 1 – Hand in before Monday, 26.09.2016 - 12.00

1. List all graphs with exactly four vertices and exactly four edges. Do not repeat isomorphic graphs and don't forget loops.

2. Show that the following is true for any graph:
   a) There is an even number of vertices whose degree is odd.
   b) A graph with n vertices and strictly more than $\frac{(n-1)(n-2)}{2}$ edges is connected.
   c) A connected graph with n vertices has at least $n-1$ edges.

3. Let V_1, V_2, ..., V_r be sets with n elements. The *complete r-partite graph* K_n^r has vertex set $V_1 \cup V_2 \cup \cdots \cup V_r$ with edge set defined by: for $i = 1, \ldots, r$, each vertex in V_i is adjacent to all other vertices except those of V_i.
   a) Draw the graphs K_2^2, K_3^2 and K_2^3.
   b) How many edges does K_n^r have?
   c) Show that K_n^r is connected for $r > 1$.
   d) For which values of r and n is K_n^r Eulerian?

4. During winter, a snow plough driver has to clean the roads of the his city, which are represented by the edges of the graph below. In order to be efficient, he wants to plough each road exactly once. How should he proceed?



5. Look up, describe and explain in your own words a practical application of graphs.