
Mathematical Methods for Computer Science II

Spring 2017

Series 12 – Hand in before Monday, 22.05.2017 - 13.00

1. Is $G = \forall x (P(x) \Leftrightarrow P(y))$ equivalent to $F = (\exists x P(x) \Leftrightarrow P(y))$? Explain?
2. i) Which of the following structures $\mathcal{A} = (\mathcal{U}_A, \mathcal{I}_A)$ are models for the formula
$$F = \forall z \exists y \forall x (P(x, y) \vee P(z, y) \vee P(x, z) \vee \neg P(z, x))?$$
 - a) $\mathcal{U}_A = \mathbb{N}$, $\mathcal{I}_A(P) = \{(m, n) \mid m, n \in \mathbb{R}, m = \alpha n, \alpha \in \mathbb{N} \setminus \{0, 1\}\}$.
 - b) $\mathcal{U}_A = \mathbb{N}$, $\mathcal{I}_A(P) = \{(2m, m) \mid m \in \mathbb{N}\}$.
 - c) $\mathcal{U}_A = 2^{\mathbb{Z}}$ (the set of all subsets of \mathbb{Z}), $\mathcal{I}_A(P) = \{(A, B) \mid A, B \subset \mathbb{Z}, A \subset B\}$.
 - ii) * Can you modify those structures which are not models so that they become a model for F and those that are models so that they no longer are models for F ?
3. Consider the following formulas F_1, F_2, F_3 which express that the predicate P is reflexive, symmetric and transitive.

$$F_1 = \forall x P(x, x)$$

$$F_2 = \forall x \forall y (P(x, y) \Rightarrow P(y, x))$$

$$F_3 = \forall x \forall y \forall z ((P(x, y) \wedge P(y, z)) \Rightarrow P(x, z))$$

Show that none of these formulas is a consequence of the other two by presenting structures which are models for two of the formulas, but not for the respective third formula.

4. For each of the formulas below, determine whether the formula is valid, satisfiable but not valid, or unsatisfiable. Explain why, and if you claim the formula is satisfiable but not valid, give interpretations to justify your answer.
 - a) $(\forall x P(x)) \Rightarrow P(f(x))$.
 - b) $(\exists x P(x)) \wedge (\exists x \neg P(f(x)))$.
 - c) $\forall x P(x) \wedge (P(x) \Rightarrow \neg(\exists z Q(z, x) \Leftarrow \forall z Q(z, x)))$.

* Exercises with a * are intended for Discrete Mathematics II students only. However, MMI II students can gain additional bonus points by attempting them.