Mathematical Methods for Computer Science I

Fall 2016

Series 12 – Hand in before Monday, 19.12.2016 - 13.00
Reminder: Wednesday, 21.12.2016: exercises session; Thursday, 22.12.2016: lecture

1. Give context-free grammars that generate the following languages.
   a) The language of admissible sequences of *if* and *else*. An if-clause can appear alone with no else-clause, or it may be balanced by exactly one else-clause.
   b) $\{w \in \{a, b, c\}^* \mid w \text{ contains at least three } b\text{'s}\}$.
   Hint: think first about the minimal number of productions needed.
   c) The set $\{a^i b^j c^k \in \{a, b, c\}^* \mid i \neq j \text{ or } j \neq k\}$, that is, the set of strings of a's followed by b's followed by c's, such that there is either a different number of a's and b's or a different number of b's and c's, or both.
   Hint: for an unequal number of a's and b's use productions that either repetitively generate an equal number of a's and b's or produce any number of a's or any number of b's and then move on to another production.

2. Let G_1 and G_2 be context-free grammars. Prove that $L(G_1) \cup L(G_2)$ is a context-free language.

3. Design a PDA which accept the following language of all strings of 0's and 1's with an unequal number of 0's and 1's.

4. Consider the language $L = \{0^k 1^k 2^k \mid k \geq 0\}$. Use the pumping lemma for context-free languages to show that L is not context-free.