Mathematical Methods for Computer Science II

Spring 2017

Series 11 – Hand in before Monday, 15.05.2017 - 13.00

1. Using resolution, show that

$$F = (\neg B \wedge \neg C \wedge D) \vee (\neg B \wedge \neg D) \vee (C \wedge D) \vee B$$

   is a tautology.

2. Among others, John is taking three courses: Logic, Combinatorics and Graph theory. He states: *At least I like one of the three courses. If I liked Combinatorics but not Graph theory, I would like Logic. Either I like both Graph theory and Logic or I do not like any of these two courses. If I liked Graph theory, I would like Combinatorics too.*
   Can we deduce that John likes all three courses?

3. List all clauses which can be obtained from applying resolution to the following clause set M

$$M = \{\{A\}, \{\neg B\}, \{\neg A, C\}, \{B, \neg C, \neg D\}, \{\neg C, D\}, \{\neg D\}\}.$$

   Is M satisfiable?

4. Let H be a finite set of formulas in CNF and let G a be a formula in DNF. Explain how resolution can be used to show that G is a consequence of H.

5. * From the examples seen so far, we notice that among the usually huge number of possible resolution steps, only a few lead to the derivation of the empty clause if the given formula is unsatisfiable. Additionally, while the resolution process proceeds, the number of clauses (and their lengths) increases further, which causes still more choices to be tried. Therefore, possibilities of improving the efficiency of the general resolution algorithm have been constructed. One of them is *unit resolution*: derive a resolvent from two clauses C_1, C_2 only if one of these clauses is a unit clause, meaning it consists of only one literal. Prove:
   a) Show that unit resolution is complete for Horn formulas, that is, unit resolution allows to conclude whether a Horn formula is satisfiable or not.
   Hint: Recall the definition of Horn formulas and compare unit resolution with the marking algorithm for Horn formulas.
   b) Show that unit resolution is not complete in the general case.
   Hint: find an counter example where unit resolution does not allow to conclude.

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* Exercises with a $\star$ are intended for Discrete Mathematics II students only. However, MMI II students can gain additional bonus points by attempting them.