Mathematical Methods for Computer Science I

Fall 2016

Series 11 – Hand in before Monday, 12.12.2016 - 13.00

1. Suppose h is the homomorphism from the alphabet $\{0, 1, 2\}$ to the alphabet $\{a, b\}$ defined by $h(0) = a, h(1) = ab$, and $h(2) = ba$.
   a) What is $h(02112)$?
   b) If L is the language $L(21^*0)$, what is $h(L)$?
   c) If L is the language $L((ba)^*a)$, what is $h^{-1}(L)$?
   d) Suppose $L = \{abaaaba\}$, that is, the language consisting of only the word $abaaaba$. What is $h^{-1}(L)$?

2. Consider the language $L = L((0 + 10)^*)$.
   (a) Construct a DFA recognizing L.
   (b) Use the DFA from a) to show that there are exactly three equivalence classes with respect to L. Give a representative for each class.

3. Consider the language $L = \{a^{i^3} \mid i \geq 0\}$.
   (a) According to the definition of equivalence w.r.t. to a language, when are two words a^{k^3} and a^{ℓ^3} equivalent over L?
   Hint: think about possible cases where $k^3 + n = k'^3$ and $\ell^3 + n = \ell'^3$, such that k' and ℓ' are integers.
   (b) Use a) and the Myhill–Nerode theorem to decide whether or not L is regular.

4. Consider the following DFA:

δ	0	1
start $\rightarrow$ q_0	q_2	q_1
q_1	q_3	q_6
q_2	q_4	q_5
q_3	q_3	q_6
q_4	q_4	q_5
((q_5))	q_5	q_2
((q_6))	q_6	q_1

   a) Represent this DFA in the form of a transition graph.
   b) Apply the minimisation algorithm seen in class to identify the equivalence classes.
   c) Give the transition table and the transition graph of the minimal DFA.