Université de Fribourg
Universität Freiburg

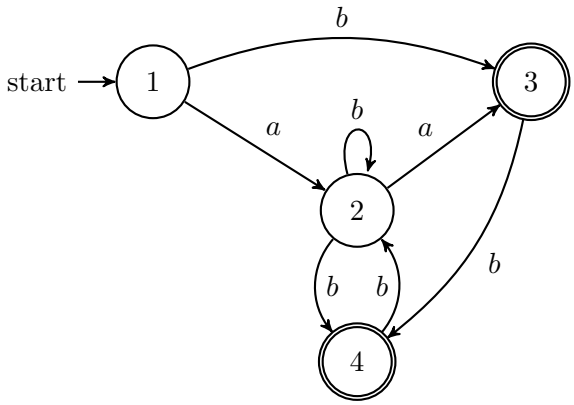
Département de Mathématiques
Mathematik Departement

Mathematical Methods for Computer Science I

Fall 2016

Series 10 – Hand in before Monday, 05.12.2016 - 13.00

1. Convert the following automaton into a generalized NFA (GNFA) and then into a regular expression.



2. Determine which of the following languages are regular. Justify your answer with a proof.
   a) $\{x1y \mid x, y \in \{0,1\}^*, |x| = 2|y|\}$.
   b) $(0+1)^*1(0+1)^*$.
   c) The set of strings of balanced parentheses. These are the strings of characters "(" and ")" that can appear in a well-formed arithmetic expression.
   d) $\{(01)^i \mid i \geq 0\}$.
   e) The set of strings of 0's and 1's that are of the form ww^R, that is, some string w followed by its reverse w^R.
   f) The set of strings of 0's and 1's that have an unequal number of 0's and 1's.
   g) * The language consisting of all strings of 1's whose length is a prime number.

3. a) Let D be a DFA with k states. Show that if $L(D)$ is infinite, then D accepts a string z with $2k \leq |z| < 3k$.
   b) Give an example of a DFA D with 4 states over the binary alphabet such that its language $L(D)$ is infinite and the shortest accepted word has length 3.

4. Prove that if L_1 and L_2 are regular languages over the same alphabet Σ, then the difference $L_1 \setminus L_2 = \{w \in L_1 | w \notin L_2\}$ is also a regular language.

5. Look up, describe and explain in your own words two practical applications of automata theory.

* Exercises with a $\star$ are intended for Discrete Mathematics I students only. However, MMI I students can gain additional bonus points by attempting them.