Université de Fribourg
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Département de Mathématiques
Mathematik Departement

Mathematical Methods for Computer Science II

Spring 2017

Outline 9

Definitions: Let G be a formula with atomic subformulas $\{A_1, A_2, \ldots\}$. We define an **assignment** (or **interpretation**) as

$$\mathcal{A} : \{A_1, A_2, \ldots\} \to \{0, 1\}.$$

If $\mathcal{A}(G) = 1$ (true), then we write $\mathcal{A} \vDash G$, and we say that F **holds under** $\mathcal{A}$ or $\mathcal{A}$ is a **model for** G. Otherwise $\mathcal{A} \nvDash G$.
A formula G is **satisfiable** if G has at least one model, otherwise G is called **unsatisfiable** (or a contradiction).
A formula G is **valid** (or a tautology) if every assignment $\mathcal{A}$ is a model for G. Notation $\vDash G$.

Theorem. *A formula F is a tautology $\Leftrightarrow \neg F$ is unsatisfiable.*

Definition: A formula is a **Horn formula** if it is in CNF and every disjunction (clause) contains at most one positive literal.

Theorem. *The "marking algorithm" is correct for Horn formulas as input, and stops after at most n applications of the while loop, where n is the number of atomic formulas in F.*

Definition: A set M of formulas is **satisfiable** if there is a **model** $\mathcal{A}$ for M, that is for every formula $F \in M$, $\mathcal{A}(F) = 1$.