Mathematical Methods for Computer Science II

Spring 2017

Outline 8

Part 4: Logic

References:
- U. Schöning, *Logic for Computer Scientists*, Birkhäuser
- S. Lipschutz, M. Lipson, *Discrete Mathematics*, Schaum's outline

4.1. **Propositional logic.**

Definitions:
- A **statement** is a sentence which is neither a question nor a command.
- A **proposition** is a statement which is either true (value 1 or True or T) or false (value 0 or False or F), but not both.

Definitions: An **atomic** or **primitive proposition** is a proposition which cannot be broken down into simpler propositions. A **compound proposition** or **propositional formula** is built by composing atomic, using the **logical connectives** "and" ($\wedge$), "or" ($\vee$), "not" ($\neg$).

Definition: Two propositional formulas f, g are **logically equivalent**, $f \equiv g$, if the formula f is true whenever g is true and vice versa; this is the case if the truth tables of f and g are the same.

Definitions: A **tautology** is a propositional formula which is always true. A **contradiction** is a propositional formula which is always false.

Definition: The statement "$p \Rightarrow q$", called **conditional**, is false only when p is true and q is false. The statement "$p \Leftrightarrow q$", called **biconditional**, is true whenever p and q have the same truth value, false otherwise.

Definitions: A **literal** is an atomic formula or the negation of an atomic formula.

(1) A formula F is in **conjunctive normal form** (CNF) if it is a conjunction of disjunctions of literals
$$F = \bigwedge_{i=1}^{n} \left(\bigvee_{j=1}^{m_i} L_{ij} \right)$$
where $L_{ij} \in \{A_1, \ldots, A_p\} \cup \{\neg A_1, \ldots, \neg A_p\}$. Each $\bigvee_{j=1}^{m_i} L_{ij}$ is called a (disjunctive) **clause**.
ex.: $\underbrace{(A_1 \vee A_2 \vee \neg A_3)}_{\text{clause}} \wedge \underbrace{(A_2 \vee A_4 \vee \neg A_5)}_{\text{clause}}$

(2) A formula F is in **disjunctive normal form** (DNF) if it is a disjunction of conjunctions of literals
$$F = \bigvee_{i=1}^{n} \left(\bigwedge_{j=1}^{m_i} L_{ij} \right).$$
Here, each $\bigwedge_{j=1}^{m_i} L_{ij}$ is called a (conjunctive) clause.

Theorem. *Every formula F can be written/transformed in CNF and in DNF.*