Université de Fribourg
Universität Freiburg

Département de Mathématiques
Mathematik Departement

Mathematical Methods for Computer Science I

Fall 2016

Outline 7

Part 2: Automata Theory and Languages

References:

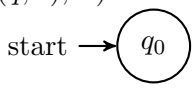
- Hopcroft, Motwani, Ullman: *Introduction to Automata Theory, Languages and Computation*
- Sipser: *Introduction to the Theory of Computation*
- Hoffmann: *Theoretische Informatik*
- Hromkovic: *Theoretische Informatik*
- Schöning: *Theoretische Informatik - kurz gefasst*, Springer

2.1. Languages.

Definitions:

- An **alphabet** is a set of symbols.
- A **string** or a **word** is a finite sequence of symbols from an alphabet.
- The **empty string** is denoted by ε.
- The **length of a string** is the number of symbols in a string.
- If Σ is an alphabet, then Σ^k is the **set of all strings of length** k with symbols of Σ.
- The **set of all strings** over an alphabet Σ is denoted by Σ^*: $\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \ldots$.
- The **set of strings of positive length** over an alphabet Σ is denoted by Σ^+: $\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \ \ldots = \Sigma^* \setminus \{\varepsilon\}$.
- The **concatenation** of strings x and y, denoted xy, is the string obtained by attaching y to x after x.
- Any set of strings from Σ^*, where Σ is an alphabet, forms a language; $L \subset \Sigma^*$ is a **language over** Σ.

Definition: A **deterministic finite automaton** (DFA) $D = (Q, \Sigma, \delta, q_0, F)$ has

- A finite set of states: Q
- A finite set of input symbols: Σ
- A transition function: $\delta : Q \times \Sigma \to Q$, i.e. δ takes as arguments a state and an input symbol and returns a state. Moreover, for all *strings* w and *symbols* a: $\delta(q, aw) = \delta(\delta(q, a), w)$.
- A start state: 

- A set of final or **accepting states**: F 

Definitions:

- A string x is **accepted** or recognized by a finite automaton $D = (Q, \Sigma, \delta, q_0, F)$ if $\delta(q_0, x) \in F$.
- The **language accepted** by D is the set $L(D) = \{x \in \Sigma^* \mid \delta(q_0, x) \in F\}$.
- A language L is called a **regular language** if it is recognized by some finite automaton, i.e. $\exists D$ automaton s.t. $L = L(D)$.