Université de Fribourg
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Département de Mathématiques
Mathematik Departement

Mathematical Methods for Computer Science I

Fall 2016

Outline 4

1.3. Matching problems.

Definition: A graph $G = (V, E)$ is **bipartite** if V can be partitioned into two sets V_1 and V_2 ($V_1 \cup V_2 = V$, $V_1 \cap V_2 = \emptyset$) such that each edge in E connects a vertex of V_1 to a vertex of V_2 (i.e. there are no edges connecting only vertices within V_1 or V_2).

Theorem. *A graph $G = (V, E)$ is bipartite $\iff$ G does not contain a cycle of odd length.*

Corollary. *Every non-trivial tree is bipartite.*

Definitions:

- A subset $M \subseteq E$ of the edges of a graph $G = (V, E)$ is called a **matching** if each vertex is incident to *at most* one edge of M.
- A matching is **perfect** if each vertex of G is incident to *exactly* one edge of M.
- A vertex in G which is incident with no edge of M is called an **unmatched vertex**.
- A matching of maximum cardinality in G is a **maximum matching**.
- An **alternating path (or circuit)** in a graph $G = (V, E)$ with respect to a matching M is a path (or circuit) $(v_1, \ldots, v_k)$ where either $(v_1, v_2), (v_3, v_4), \ldots$ are in M or $(v_2, v_3), (v_4, v_5), \ldots$ are in M.
- An **augmenting path** P is an alternating path in which *both* end vertices are unmatched to M.

Theorem (Berge, 1957)**.** *Let M be a matching in any graph $G = (V, E)$.*
M can be extended to M' $\iff$ there is an augmenting path in G with respect to M.