Mathematical Methods for Computer Science II

Spring 2017

Outline 3

3.3. Generating functions.

Definition: The **generating function (GF)** of a sequence of numbers $(a_k)_{k\in\mathbb{N}} = (a_0, a_1, \ldots)$ is the formal power series

$$f(z) = \sum_{k=0}^{\infty} a_k z^k = a_0 + a_1 z + a_2 z^2 + \ldots.$$

Theorem. *Let $A_1, \ldots, A_n$ be subsets of $\mathbb{N}$. For each k, let a_k be the number of non-negative integer solutions $(x_1, ..., x_n) \in \mathbb{N}^n$ of the equation*

$$x_1 + \ldots + x_n = k$$

subject to the restriction $x_i \in A_i$. The generating function of the sequence $(a_k)_{k\in\mathbb{N}}$ is

$$\sum_{k=0}^{\infty} a_k z^k = \prod_{i=1}^{n} \left(\sum_{k\in A_i} z^k \right) = \left(\sum_{k\in A_1} z^k \right) \cdot \ldots \cdot \left(\sum_{k\in A_n} z^k \right) \qquad (|z| < 1)$$