
Mathematical Methods for Computer Science I

Fall 2016

Outline 3

Definition: For a connected graph $G = (V, E)$, a **spanning tree** $T = (V, E(T))$ of G is a tree which contains every vertex of G , and $E(T) \subseteq E$.

Proposition 1. *The vertices of a tree T can be enumerated as v_1, \dots, v_n so that every v_i with $i \geq 2$ has a unique neighbour in $\{v_1, \dots, v_{i-1}\}$.*

Proposition 2. *Every connected graph contains a spanning tree.*

Proposition 3. *Let G be a connected graph with n vertices.
 G is a tree $\iff G$ has $n - 1$ edges.*

Definition: Let $G = (V, E)$ be a connected graph with $c : E \rightarrow \mathbb{R}^+$, a **cost function** or weight function. Such a graph is called a **weighted graph**.

Theorem. *Kruskal's algorithm produces a minimum spanning tree in a connected graph.*