Mathematical Methods for Computer Science I

Fall 2016

Outline 2

Definition: A **subgraph** of a graph $G = (V, E)$ is a graph $G' = (V, E')$ with $E' \subseteq E$. An **induced subgraph** of G is a graph $G'' = (V'', E'')$ in which $V'' \subseteq V$ and E'' contains all edges in E between two vertices of V''.

Definition: $G = (V, E)$ and $G' = (V', E')$ are **isomorphic**, written $G \cong G'$ if both

·) there exists a bijection (isomorphism) $\varphi : V \to V'$
·) (x, y) is an edge in G exactly when $(\varphi(x), \varphi(y))$ is an edge in G'.

Theorem (Hakimi)**.** *A sequence of numbers $d_1, d_2, \ldots, d_n$ with $d_i \geq 0, d_1 \geq d_2 \geq \ldots \geq d_n$ represents the degrees of vertices in a graph $\iff d_2 - 1, d_3 - 1, \ldots, d_{d_1+1} - 1, d_{d_1+2}, \ldots, d_n$ represent the degrees of vertices in a graph.*

Definition: A graph is called a **Hamiltonian graph** if it has a circuit through all its vertices. Such a circuit is called a **Hamiltonian circuit**.

Theorem. *If $G = (V, E)$ is a graph with $|V| = n \geq 3$ such that for all distinct non-adjacent vertices u and v, $d(u) + d(v) \geq n$, then G is Hamiltonian.*

1.2. **Trees.**

Definition:

- A **tree** is a connected graph without circuits.
- A **forest** is a graph whose components are trees.
- The vertices of degree 1 in a tree are called **leaves**.

Theorem. *The following statements are equivalent:*

(1) T is a tree.
(2) There is a unique path between any two vertices in T.
(3) T is connected but $T \backslash e$ is disconnected for every edge e in T.
(4) T contains no circuit but $T \cup \{(x, y)\}$ does for any non-adjacent vertices x, y in T.