
Mathematical Methods for Computer Science I

Fall 2016

Outline 1

Part 1: Graph Theory

References:

- Chartrand, Lesniak, Zhang: *Graphs & Digraphs*, Chapman & Hall
- Harris, Hirst, Mossinghoff: *Combinatorics and Graph Theory*, Springer
- Teschl: *Mathematik für Informatiker*, Band 1, Springer
- Bollobás: *Modern Graph Theory*, Springer
- Diestel: *Graph Theory*, Springer

1.1. Graphs.

Definition: A **graph** G is a pair (V, E) , where $V = \{v_1, \dots, v_n\}$ is a set of vertices (Ecken, sommets) or nodes (Knoten, noeuds) and E is a set of edges (Kanten, arêtes). Each edge is represented by a pair of vertices. A graph is **(un)directed** if the pairs of vertices representing the edges are (un)ordered. A directed graph is also called a digraph.

Definitions:

- A vertex v and an edge e are **incident** if $v \in e$.
- Two vertices v, w are **adjacent** (or **neighbouring**) if $(v, w) \in E$.
- The **degree** $d(v)$ of a vertex v is the number of edges incident with v .
- The **minimal degree** of the vertices of a graph G is denoted by $\delta(G)$.
- The **maximal degree** of the vertices of a graph G is denoted by $\Delta(G)$.

Proposition. For any graph $G = (V, E)$,

$$\sum_{v \in V} d(v) = 2|E|.$$

Definitions:

- A graph whose vertices are all pairwise adjacent is a **complete graph**.
- A **walk** is an alternating sequence of incident vertices and edges.
- A **path** is a walk where all vertices are distinct.
- A **cycle** is a closed walk (not necessarily a path).
- A **circuit** is a closed path.
- The **length** of a walk is given by the number of edges in it.
- A graph G is **connected** if for every pair $\{x, y\}$ of distinct vertices there is a path (not necessarily an edge) from x to y .
- For a graph $G = (V, E)$, the **adjacency matrix** $A(G)$ is an $n \times n$ matrix $A(G) = (a_{ij})_{\substack{1 \leq i \leq n \\ 1 \leq j \leq n}}$ such that

$$a_{ij} = \begin{cases} 1 & \text{if } (v_i, v_j) \in E \\ 0 & \text{if } (v_i, v_j) \notin E. \end{cases}$$

Theorem. If A is the adjacency matrix of a graph, then the (i, j) entry of A^k , $k \geq 1$, is the number of different walks of length k between v_i and v_j .

Definition: A graph is called an **Eulerian graph** if it contains a cycle in which every edge of the graph is traversed exactly once. We call this cycle an **Eulerian cycle**.

Theorem. A connected graph G is Eulerian \iff every vertex has even degree.