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## Mathematical Methods for Computer Science II

Spring 2017

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### Outline 13

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**Definition:** A formula is **rectified** if no variable occurs both bound and free and if all quantifiers in the formula refer to different variables.

**Lemma.** *Every formula  $F$  can be transformed into an equivalent rectified formula via substitutions of variables.*

**Definition:** A formula is in **prenex form** if it has the form  $Q_1y_1Q_2y_2 \dots Q_ny_nF$ , where  $Q_i \in \{\forall, \exists\}$  and the  $y_i$  are variables.  $F$  does not contain a quantifier.

**Theorem.** *For every formula  $F$ , there exists an equivalent and rectified formula  $G$  in prenex form, called RPF.*

**Definition:** A **Skolem form** is a prenex rectified formula without existential quantifier ( $\exists$ ).

**Theorem.** *Let  $F$  be a prenex rectifiable formula.*

$F$  is satisfiable  $\Leftrightarrow$  the Skolem form of  $F$  is satisfiable.

### Undecidability

**Definition:** A function is called **computable** (or a problem is called **decidable**) if there exists an automaton (Turing machine) which, started with an input in the function domain, halts after a finite number of steps and outputs the correct function value. Otherwise, the function (problem) is called **non-computable** (**undecidable**).

**Theorem** (Church). *The validity problem for formulas in predicate logic is undecidable.*

**Corollary.** *The satisfiability problem in predicate logic is undecidable.*