Université de Fribourg
Universität Freiburg

Département de Mathématiques
Mathematik Departement

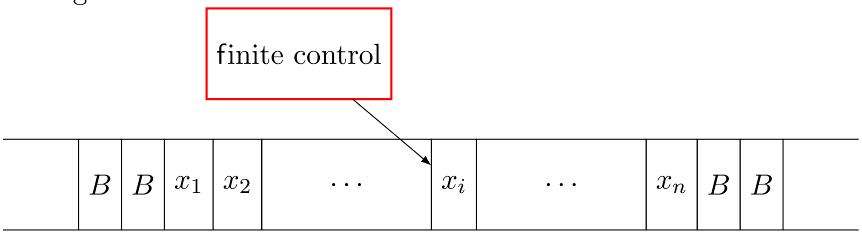
Mathematical Methods for Computer Science I

Fall 2016

Outline 13

2.10. **Introduction to Turing Machines (TM).**

Turing machine:



(1) finite control: is in any of a finite set of states
(2) infinite tape: divided into cells, each with a finite number of symbols, or a blank "B".

Formal notations: $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$
Q: states
Σ: alphabet
Γ: tape symbols
δ: transition function : $\delta(q, X) = (p, Y, D)$ where q and p are states, Y replaces the tape symbol X from the cell being scanned, D is a direction (left/right)
q_0: start state; B: blank symbol; F: set of accepting states.

Definition: Let $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ be a Turing Machine. $L(M)$, the **language accepted by the TM** M, is $\{w \in \Sigma^* | q_0 w \vdash^* \alpha p \beta$ for some $p \in F$ and $\alpha, \beta \in \Gamma\}$. The set of languages for which there exists a TM is called the set of **recursively enumerable languages**.

Definition: The languages with Turing machines that *halt* eventually, whether or not they accept, are called **recursive**.

Definition: A problem is **decidable** (the corresponding language is recursive) if it has an algorithm that always tells correctly whether an instance of the problem has answer *yes* or *no*. Otherwise the problem is **undecidable**.

Theorem. *If a language L is recursive, then its complement $\bar{L}$ is also recursive.*

Theorem. *If both a language L and its complement are recursively enumerable, then L is recursive.*

Theorem (Rice). *Every nontrivial property of the recursively enumerable languages is undecidable.*

Definition: The classes $\mathcal{P}$ and $\mathcal{NP}$ of problems are those that are solvable in polynomial time by deterministic and nondeterministic TMs. Problems in $\mathcal{NP}$ are also called intractable. This definition follows the unproven assumption that $\mathcal{P} \neq \mathcal{NP}$.