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Mathematical Methods for Computer Science II

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Outline 11

Definition: Let F be a set of clauses. Then

$$Res(F) = F \cup \{R \,|\, \text{R is a resolvent of two clauses in } F\}.$$

Furthermore:

$$\begin{aligned} Res^0(F) &= F \\ Res^{n+1}(F) &= Res(Res^n(F)) \qquad \text{for } n \geq 0 \\ Res^*(F) &= \bigcup_{n\geq 0} Res^n(F). \end{aligned}$$

Theorem (Resolution Theorem of propositional logic)**.**
A clause set F is unsatisfiable $\iff \Box \in Res^*(F)$.

Definition: A **derivation** of the empty clause from a clause set F is a sequence $C_1, C_2, \dots, C_m$ of clauses such that C_m is the empty clause, and for every $i = 1, \dots, m$, C_i either is a clause in F or a resolvent of two clauses C_a, C_b with $a, b < i$.

Lemma. *A clause set F is unsatisfiable $\iff$ a derivation of the empty clause from F exists.*

Definition: A formula G is a **consequence** of a set of formulas $\{F_1, \dots, F_k\}$ if for every interpretation $\mathcal{A}$, whenever $\mathcal{A}(F_i) = 1$ for all i, $\mathcal{A}(G) = 1$.

Lemma. *If every member of a set of clauses contains a negative literal, then that set of clauses is satisfiable.*

4.2. Predicate logic.

Definition: (first-order) predicate logic is a formal system with the following ingredients:

1) Lexicon with:
   a) set of **constants** $(a, b, c, \dots)$
   b) set of **variables** $(x, u, v, w, \dots)$
   c) set of **function symbols** $(f, g, h, \dots)$
   d) set of **predicate symbols** $(P, Q, \dots)$
   e) two **quantifiers** $\forall$ and $\exists$
   f) logical **connectives** $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$
   g) **punctuation marks** "(", ")", ",", ...
2) Rules of formation of formulas in predicate logic:

·) **terms**: 1. Each variable is a term
2. If f is a function with arity k and if $t_1, \dots, t_k$ are terms, then $f(t_1, \dots, t_k)$ is a term. A function or predicate has **arity** k if it depends on k objects.

·) **formulas**: 1. If P is a predicate with arity k and if $t_1, \dots, t_k$ are terms, then $P(t_1, \dots, t_k)$ is a formula.
2. For all formulas F and G, $\neg F$, $F \wedge G$ and $F \vee G$ are formulas.
3. If x is a variable and F is a formula, then $\exists x F$ and $\forall x F$ are formulas.

Definition: An occurrence of the variable x in a formula G is **bound** if x occurs within a subformula of G of the form $(\exists x)G$ or $(\forall x)G$. Otherwise the variable has free occurrence. A formula without occurrence of a free variable is called **closed**.