Mathematical Methods for Computer Science I

Fall 2016

Outline 11

2.7. Equivalence and minimisation.

Definition: Let L be a language over an alphabet Σ. $u, v \in \Sigma^*$ are **equivalent** or **indistinguishable** w.r.t. L if for every $w \in \Sigma^*$ either $\begin{cases} uw \in L \\ vw \in L \end{cases}$ or $\begin{cases} uw \notin L \\ vw \notin L \end{cases}$.
Notation: $u \equiv_L v$

Lemma. *For any language L, $\equiv_L$ is an equivalence relation:*
1) $u \equiv_L u$ (reflexivity)
2) $u \equiv_L v \implies v \equiv_L u$ (symmetry)
3) $u \equiv_L v$ and $v \equiv_L w \implies u \equiv_L w$ (transitivity)

Definition: Let L be regular and D a DFA s.t. $L = L(D)$; $D = (Q, \Sigma, \delta, q_0, F)$.
$u, v \in \Sigma^*$ are **equivalent w.r.t. D**, $u \equiv_D v$, if $\delta(q_0, u) = \delta(q_0, v)$.

Theorem (Myhill–Nerode)**.** *Let L be a language over an alphabet Σ. Then L is regular $\iff \equiv_L$ has a finite number of equivalence classes.*

Corollary. *For every regular language L there is a minimal DFA accepting L. Its number of states is the number of equivalence classes w.r.t. $\equiv_L$. The minimal DFA is unique up to renaming of the states. This DFA is given by the construction in " $\Leftarrow$ " in the proof of the Myhill–Nerode Theorem.*