Université de Fribourg — Département de Mathématiques
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Mathematical Methods for Computer Science II

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Outline 10

Definition: A set M of formulas is **satisfiable** if there is a **model** $\mathcal{A}$ for M, that is for every formula $F \in M$, $\mathcal{A}(F) = 1$.

Theorem (Compactness Theorem)**.** *A set M of formulas is satisfiable $\Leftrightarrow$ every finite subset of M is satisfiable.*

Corollary. *A set of formulas M is unsatisfiable $\Leftrightarrow$ there is a finite subset of M which is unsatisfiable.*

Definition: Let C_1, C_2, R be clauses. Then R is called a **resolvent** of C_1 and C_2 if there is a literal $L \in C_1$ such that $\overline{L} \in C_2$ and R has the form

$$R = (C_1 \backslash \{L\}) \cup (C_2 \backslash \{\overline{L}\}),$$

where

$$\overline{L} = \begin{cases} \neg A_i & \text{if } L = A_i, \\ A_i & \text{if } L = \neg A_i. \end{cases}$$

Lemma (Resolution Lemma)**.** *Let F be a formula in CNF, represented as a set of clauses. Let R be a resolvent of two clauses C_1 and C_2 in F.*
Then $F \equiv F \cup \{R\} \equiv ((\dots) \wedge (\dots) \wedge \dots \wedge (\dots) \underline{\wedge} R)$ in the sense that a model $\mathcal{A}$ for F is also a model for $F \cup \{R\}$ and vice versa.