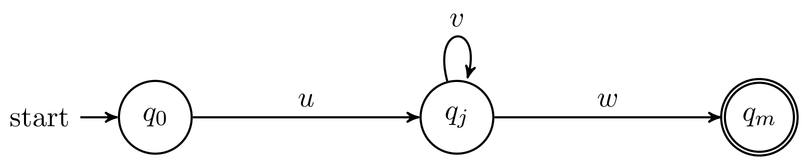
Mathematical Methods for Computer Science I

Fall 2016

Outline 10

2.5. Properties of regular languages.

Lemma (The Pumping Lemma).
Let L be a regular language which is recognized by a DFA with n states.
Any string z in L with $|z| \geq n$ can be written as $z = uvw$ with $|uv| \leq n$, $|v| > 0$ such that $uv^iw \in L$ for all $i \geq 0$.



Examples:
1) The language $L = \{0^i1^i \mid i \geq 0\}$ is not regular.
2) The language $L = \{z \in \{a,b\}^* \mid |z| \text{ is a perfect square } (1,4,9,16,25,\ldots)\}$ is not regular.

Theorem. *Let D be a DFA with k states*
i) $L(D) \neq \emptyset \iff D$ accepts a string z with $|z| < k$
ii) $L(D)$ is infinite $\iff D$ accepts a string z with $k \leq |z| < 2k$.

2.6. Closure properties of regular languages.

Theorem. *Regular languages are closed under union, concatenation and star (Kleene) closure.*

Theorem.
1) *Regular languages are closed under taking complements; that is if L is regular over an alphabet Σ, then $\Sigma^* \backslash L$ is also regular.*
2) *Regular languages are closed under intersection.*

Definition: Let Σ_1 and Σ_2 be two alphabets. $h : \Sigma_1^* \to \Sigma_2^*, a_1a_2\ldots a_n \mapsto h(a_1\ldots a_n) = h(a_1)h(a_2)\ldots h(a_n), h(\varepsilon) = \varepsilon$ is called a **homomorphism** or **substitution**, i.e. h substitutes a particular string $h(a_i) \in \Sigma_2^*$ for each symbol $a_i \in \Sigma_1$.

Theorem. *Regular languages are closed under homomorphism. That means: if L is regular and h is a homomorphism, then $h(L)$ is regular. Formally if $L = L(R)$ for a RE R, then $L(h(R)) = h(L(R))$.*

Definition: Let Σ_1 and Σ_2 be two alphabets and $h : \Sigma_1^* \to \Sigma_2^*$ a homomorphism. The set of strings w in Σ_1^* s.t. $h(w)$ is in Σ_2^* is called the **inverse homomorphism**, denoted by h^{-1}.
Notice that for a string v in Σ_2^*, $h^{-1}(v) = \{w \in \Sigma_1^* \mid h(w) = v\} \subseteq \Sigma_1^*$ is a *set*.

Theorem. *Regular languages are closed under inverse homomorphism.*